Representation Geometry of the Turbulent World

Evidence from cross-Reynolds transfer in 2D forced Navier-Stokes

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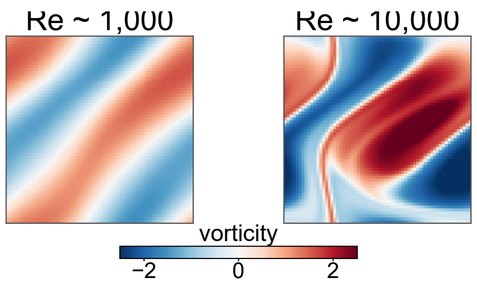
arXiv

Across a 10x Reynolds shift, transfer error follows **state-space geometry** more closely than the choice between learned and retrieved dynamics.

Pursuing Cross-Reynolds Generalisation



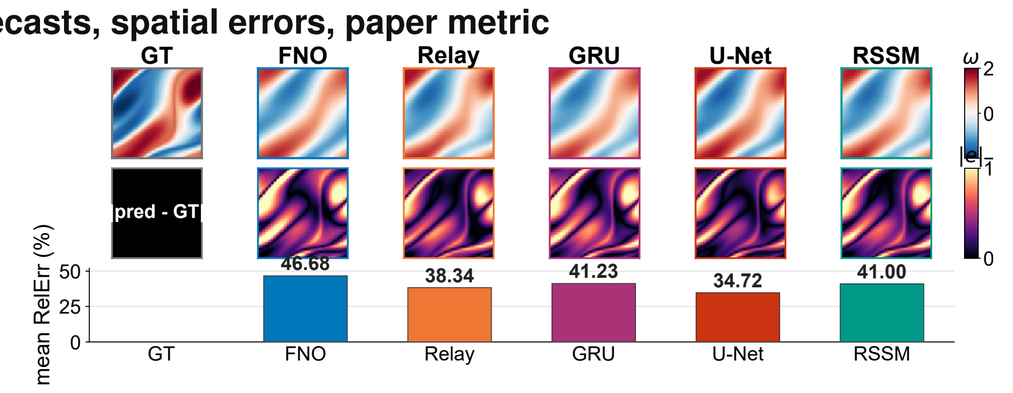
A 10x Reynolds shift

Train: Re approx. 1,000 **Test:** Re approx. 10,000

2D forced Navier-Stokes, 64 x 64 periodic, forcing wave number kf = 4.

10 observed frames to 10 predicted frames. N = 500 trajectories, 3 seeds; 95% trajectory bootstrap CIs.

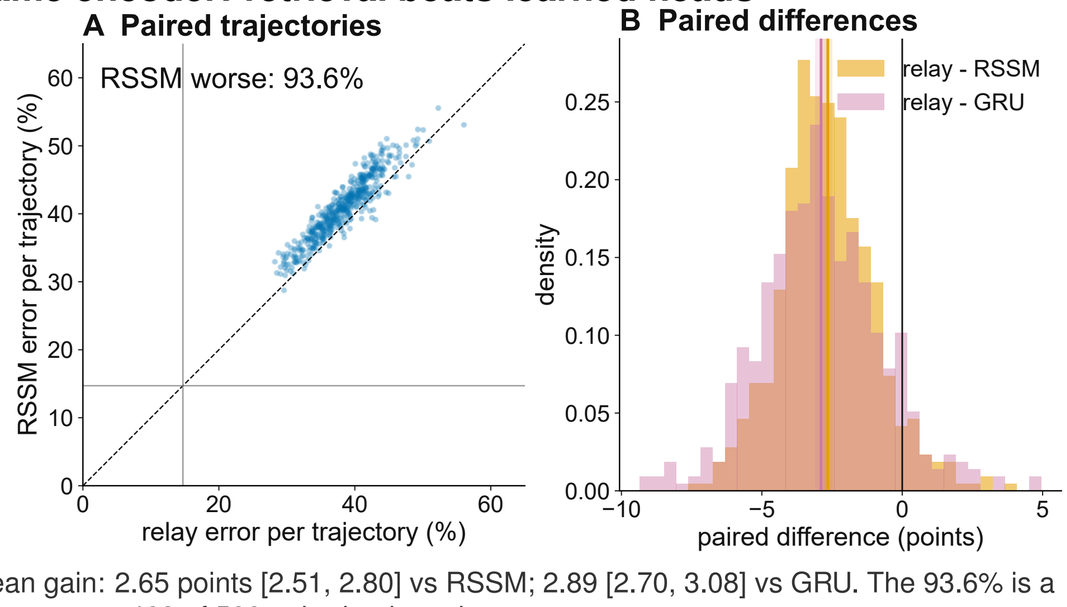
Forecasts, spatial errors, paper metric



Trajectory 24, step 6, seed 42 fields at Re approx. 10,000. Middle: absolute error on one shared scale. Bars: audited N = 500 x 10-step means, not snapshot errors.

An Unexpected Result

Same encoder: retrieval beats learned heads



Mean gain: 2.65 points [2.51, 2.80] vs RSSM; 2.89 [2.70, 3.08] vs GRU. The 93.6% is a direct count: 468 of 500 paired trajectories.

Current results: size does not rank transfer

Method	Params	Representation	RelErr	95% CI
U-Net	7.80M	local multi-scale	**34.72**	[34.20, 35.22]
ConvAE-Relay	1.60M	local 64-d	38.34	[37.96, 38.73]
RSSM-style	5.27M	local 64-d	41.00	[40.60, 41.40]
kNN-copy	0	raw field 4096-d	41.01	[40.52, 41.50]
ConvAE-GRU	1.71M	local 64-d	41.23	[40.90, 41.55]
PCA relay	0	global linear 36-d	42.35	[41.79, 42.92]
FNO	0.47M	global spectral	46.68	[46.19, 47.15]
Persistence	0	raw field	61.08	[60.23, 61.95]

N = 500 trajectories; 95% trajectory-bootstrap CIs. PCA uses the complete N = 500 rerun. U-Net leads overall; Relay is the best retrieval method.

A 5.27M-parameter learned head and a zero-parameter nearest-neighbour copy are statistically indistinguishable: 0.01 points [-0.36, 0.39].

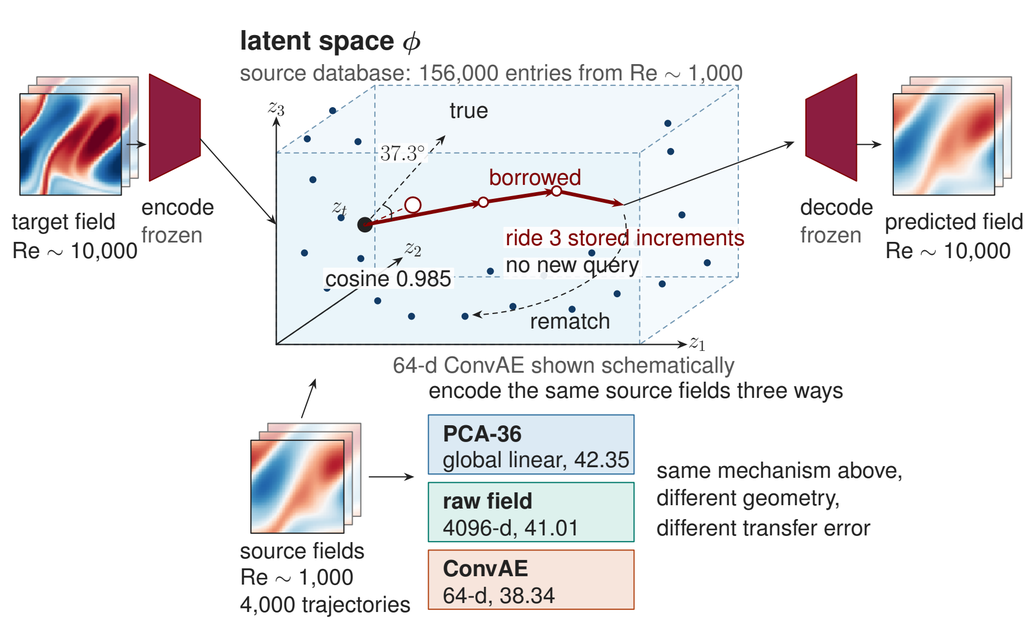
Relay: How Borrowing Works and Where It Breaks

Relay pseudocode

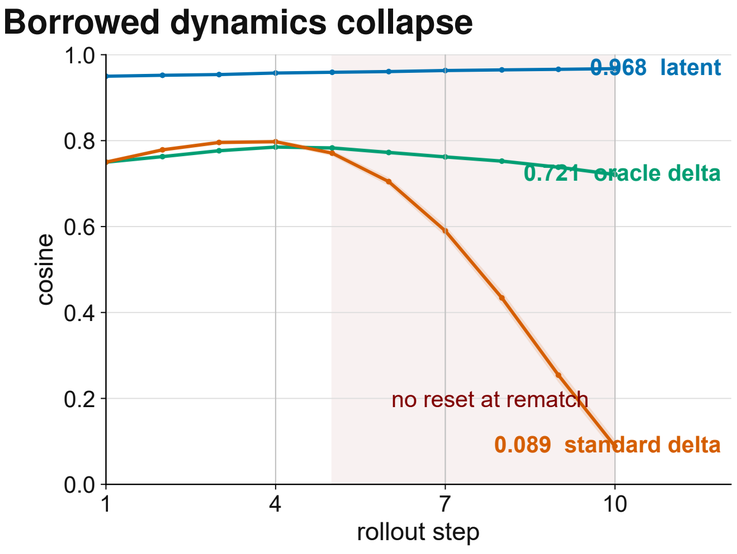
Require: $\mathcal{D} = \{(z_{j,s}, \Delta z_{j,s})\}$
Require: target ω_0, horizon T, ride $N = 3$

$z \leftarrow \phi(\omega_0)$
for $t = 1, \ldots, T$ **do**
  if $t \bmod N = 1$ **then**
    $(j, s) \leftarrow \arg\max_{(j',s')} \cos(z, z_{j',s'})$
  end if
  $\delta \leftarrow (t-1) \bmod N$
  $z \leftarrow z + \Delta z_{j,s+\delta}$
  $\hat{\omega}_t \leftarrow \phi^{-1}(z)$
end for

ϕ: source-trained ConvAE. $\mathcal{D}$ stores source latent states and next-step increments.



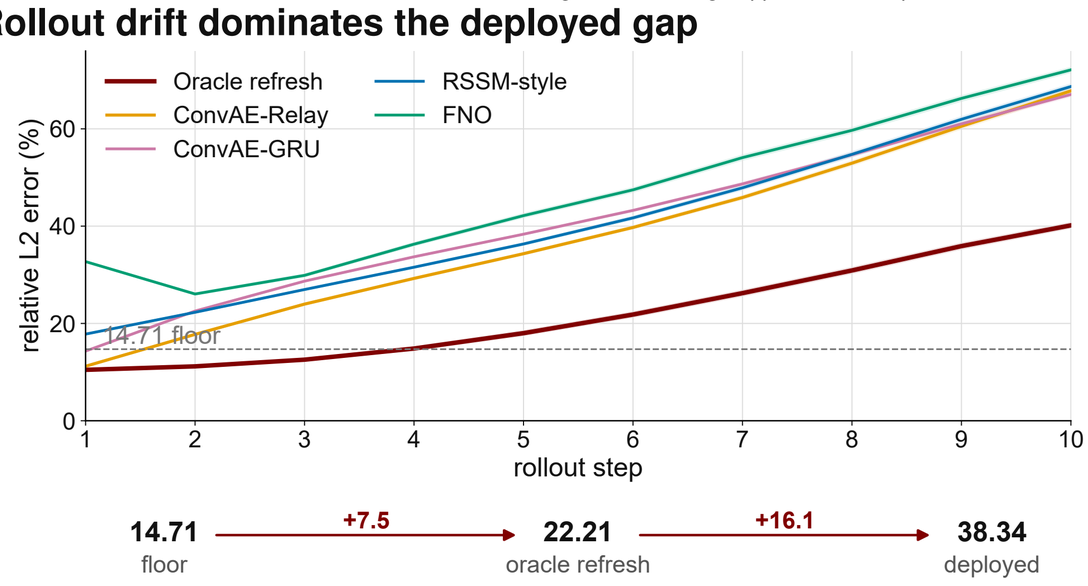
Borrowed dynamics collapse



Latent alignment improves, but the standard borrowed delta collapses. Oracle matching retains a usable direction.

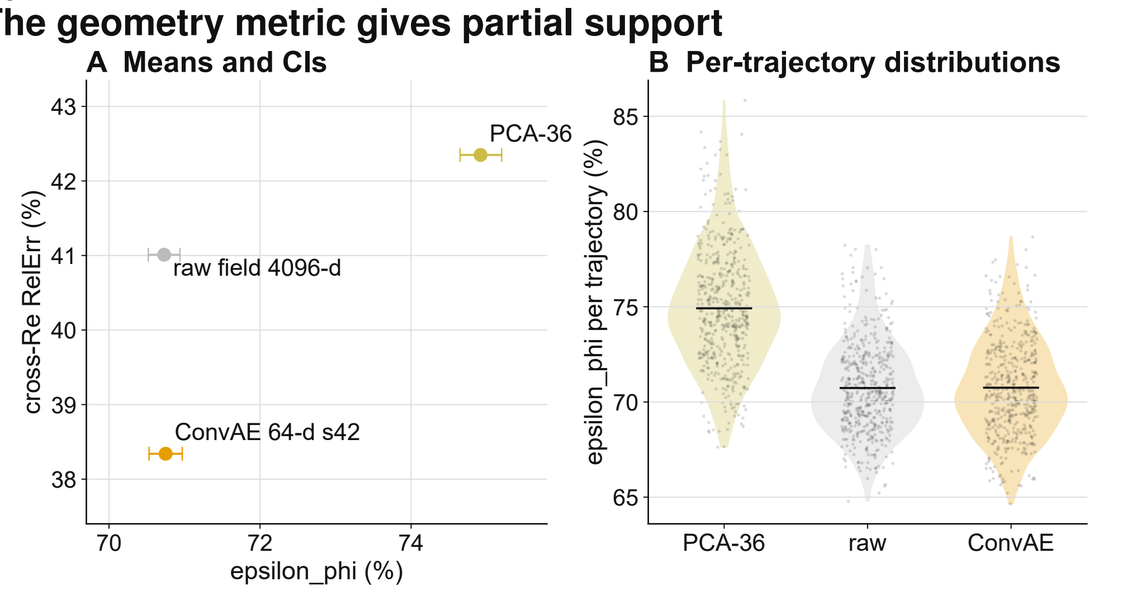
Matching and borrowing happen in latent space. The increment crosses regimes; the state never does.

Rollout drift dominates the deployed gap



Successive levels use the same encoder, decoder and evaluation set. Annotated gaps are differences between levels, not independent additive error sources.

The geometry metric gives partial support



PCA separates; raw field and ConvAE do not. The metric is informative, but not sufficient. PCA is the complete N = 500 rerun.

Where We Stand

Rematching: no measurable change

N_RIDE	RelErr (%)	95% CI
1	38.31	[37.92, 38.68]
2	38.30	[37.91, 38.68]
3	**38.34**	[37.96, 38.72]
5	38.31	[37.92, 38.68]
10	38.28	[37.90, 38.67]

Mean range = 0.06 points; all confidence intervals overlap.

Scope: 2D Kolmogorov flow only. No Kuramoto-Sivashinsky experiment; no claim of Reynolds-invariant physics. Setup fields are seed-42 snapshots; bars use the N = 500 x 10-step protocol mean.

$\cos_{\text{state}}$: 0.950 ↗ 0.968 $\cos_{\text{evolution}}$: 0.798 ↘ 0.089

Nearest in state need not be nearest in evolution.

- Under a strictly controlled swap, same frozen encoder and decoder, the transition rule moves error by 2.9 points. Across architectures the spread is 12 points and orders by representation type, not by parameter count.
- The bottleneck is finding, not borrowing. Rematching does not help; oracle matching does.
- Our geometry metric confirms this only in part. Measuring representation geometry directly is the open problem.

What this means for us: the state metric we use has no explicit notion of phase. Stay tuned.

The ordering reverses at 100x